Algebra I/2 Learning Progression

Summary of Year

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The units deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra I

- Solving characteristic problems involving the analytic geometry of lines, including, writing the equation of a line given a point and a slope.
- Adding, subtracting and multiplying polynomials.
- Transforming expressions and chunking (seeing the parts of an expression as a single object) as used in factoring, completing the square, and other algebraic calculations.

Unit 1: Growth Mindset
Unit 2: Introduction to Statistics
Unit 3: Introduction to Variables & Equations
Unit 4: Linear Equations
Unit 5: Statistics
Unit 6: Linear Equations Extension
Unit 7: Function Notation
Unit 8: Systems of Equations
Unit 9: Polynomial Algebraic Operations – The Sequel
Unit 10: Functions that Curve: Quadratics & Exponential

Updated 6.1.15

WSLS (CCSS) Major Emphasis Clusters

Seeing Structure in Expressions
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions
- Perform arithmetic operations on polynomials

Create Equations
- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Represent and solve equations and inequalities graphically

Interpreting Functions
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Interpreting categorical and quantitative data
- Summarize, represent, and interpret data on a single count or measurable variable
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models

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Rationale for Unit Sequence in Algebra I/2

Unit 1: This unit is designed to create a cohesive classroom environment that fosters the growth mindset and grit through problem solving while assessing and addressing gaps in mathematical background.

Unit 2: This unit is focused on the description of data as charts and central tendency, and accurate creation of these charts through correct meaning and use of scale.

Unit 3: The purpose of this unit is to develop application, not memorization, of algebraic properties through the review of the introductory topics of variables and equations, and basic algebraic manipulation and equalities. Focus topics include: distributive property, the difference between solve and simplify, the difference between expressions and equations, combining like terms, inverse operations, and solving for a single variable. Your next unit will cover linear equations in depth.

Unit 4: The purpose of this unit is to interpret and apply linear equations, including linear modeling. This unit will be the heart of linear algebra instruction. Students will learn how to create 1 and 2 variable equations and expressions through various strategies including investigation of multiple representations (patterns, rules, graphs, equations, tables). Students will master substitution and evaluation, solution of 1-and 2-variable equations and the order of operation.

Unit 5: This unit will extend students' application and knowledge of statistics to data which can be modeled linearly through a scatter plot and line of fit. Students will interpret data and statistical data description in context. (Think real-life application)

Unit 6: This unit is an extension of unit 3 and 4, intended to develop and support mastery of linear algebra, modeling, writing equations, and balancing and rearranging equations.

Unit 7: The purpose of this unit is to incorporate and transition student's understanding of linear relationships to the conceptual idea of functions (inputs and outputs), as well as extending it to absolute value. Students will continue to develop practical application by interpreting input and output in context. Students will also gain proficiency with absolute value, including solution and graphing.

Unit 8: This unit will build upon students' middle school exposure to graphing systems and their understanding of linear equations from earlier units. Students will extend their ability with linear modeling and algebraic equations to write and solve the following: linear systems, linear inequalities, and systems of linear inequalities.

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Unit 9: In this unit students will gain an advanced understanding of algebra skills, such as exponent rules, radical simplification, multiplying binomial and trinomial expressions, and factoring quadratic expressions. Using multiple representations, including the area model, will be important for student mastery of these topics and success in later algebra courses.

Unit 10: This unit will explore quadratic and exponential functions through multiple representations and comparison to linear and absolute value functions and graphs.

Alignment Chart

Unit and Approximate Number of Instructional Days	Washington State Learning Standards Addressed in Algebra I/2 Units
Unit 1: **Growth Mindset** (14 days) SMP 1,2,3,4,5,6,7,8	**Reason quantitatively and use units to solve problems.** **N-Q.1** Use units as a way to understand problems and to guide the solution ~~of multi-step problems~~; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Use properties of rational and irrational numbers** **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **Construct and compare linear, quadratic, and exponential models and solve problems**

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	F-LE.2 Construct linear ~~and exponential~~ functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Interpret functions that arise in applications in terms of the context.** **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
Unit 2: **Introduction to Statistics** (10 days) SMP 1,2,3,4,5,6,7,8	**Reason quantitatively and use units to solve problems.** **N-Q.1** Use units as a way to understand problems and to guide the solution ~~of multi-step problems~~; choose and interpret units consistently ~~in formulas~~; choose and interpret the scale and the origin in graphs and data displays. **Summarize, represent, and interpret data on a single count or measurement variable** **S-ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).★ **S-ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★ **S-ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★
Unit 3: **Introduction to Variables & Equations** (10 days) SMP 1,2,3,4,5,6,7,8	**Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. **A-SSE.2** Use the structure of an expression to identify ways to rewrite it. ~~*For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as*~~

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	~~$(x^2 - y^2)(x^2 + y^2)$.~~ **Understand solving equations as a process of reasoning and explain the reasoning** **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Perform arithmetic operations on polynomials** **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Create equations that describe numbers or relationships** **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*⋆
Unit 4: **Linear Equations** (25 days) SMP 1,2,3,4,5,6,7,8	**Reason quantitatively and use units to solve problems.** **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.⋆ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)_n$ as the product of P and a factor not depending on P.* **Create equations that describe numbers or relationships** **A-CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, ~~simple rational~~, and exponential functions.*⋆

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	A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*⋆ **Solve equations and inequalities in one variable** **A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Represent and solve equations and inequalities graphically** **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **Understand the concept of a function and use function notation** **F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1), f(n+1) = f(n) + f(n-1) for n ≥ 1.* **Interpret functions that arise in applications in terms of the context.**

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	F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **Analyze functions using different representations** **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Build a function that models a relationship between two quantities** **F-BF.1** Write a function that describes a relationship between two quantities.★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

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	F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Unit 5: Statistics (15 days) SMP 1,2,3,4,5,6,7,8	**Reason quantitatively and use units to solve problems.** **N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* **Interpret functions that arise in applications in terms of the context** **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

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	F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Interpret expressions for functions in terms of the situation they model** **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context. **Summarize, represent, and interpret data on two categorical and quantitative variables** **S-ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ **S-ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* c. Fit a linear function for a scatter plot that suggests a linear association.. **Interpret linear models** **S-ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ **S-ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.★ **S-ID.9** Distinguish between correlation and causation.★
Unit 6: **Linear Equations Extension** (10 days)	**Reason quantitatively and use units to solve problems.** **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

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SMP 1,2,3,4,5,6,7,8	**Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)_n$ as the product of P and a factor not depending on P.* **Create equations that describe numbers or relationships** **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★ **Understand the concept of a function and use function notation** **F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. **Interpret functions that arise in applications in terms of the context** **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

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	Build a function that models a relationship between two quantities **F-BF.1** Write a function that describes a relationship between two quantities.★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Interpret expressions for functions in terms of the situation they model** **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
Unit 7: **Function Notation** (10 days) SMP 1,2,3,4,5,6,7,8	**Understand the concept of a function and use function notation** **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Build new functions from existing functions** **F-BF.3** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

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Unit 8: **Systems of Equations** (20-25 days) SMP 1,2,3,4,5,6,7,8	**Create equations that describe numbers or relationships** **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*⋆ **Solve equations and inequalities in one variable** **A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Solve systems of equations** **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A-REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **A-REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. **Represent and solve equations and inequalities graphically** **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.⋆

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	A-REI.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Unit 9: **Polynomial Algebraic Operations – The Sequel** (20 days) SMP 1,2,3,4,5,6,7,8	**Extend the properties of exponents to rational exponents** **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Write expressions in equivalent forms to solve problems** **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ *c.* Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **Solve equations and inequalities in one variable** **A-REI.4** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

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	Analyze functions using different representations **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
Unit 10: **Functions that Curve:** **Quadratics & Exponential** (15 - 20 days) SMP 1,2,3,4,5,6,7,8	**Write expressions in equivalent forms to solve problems** **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.⋆ a. Factor a quadratic expression to reveal the zeros of the function it defines. **Understand the relationship between zeros and factors of polynomials** **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Analyze functions using different representations** **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

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Build a function that models a relationship between two quantities

F-BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

Build new functions from existing functions

F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

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	Interpret expressions for functions in terms of the situation they model **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context.